Programming paradigms

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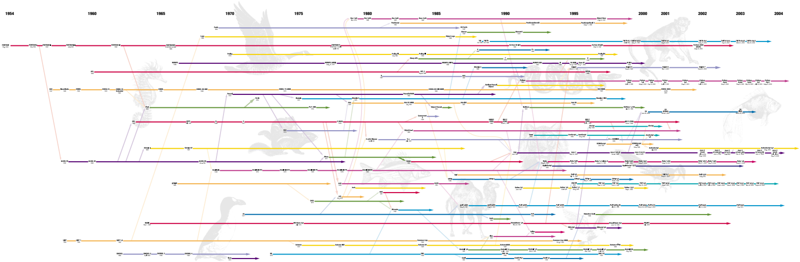
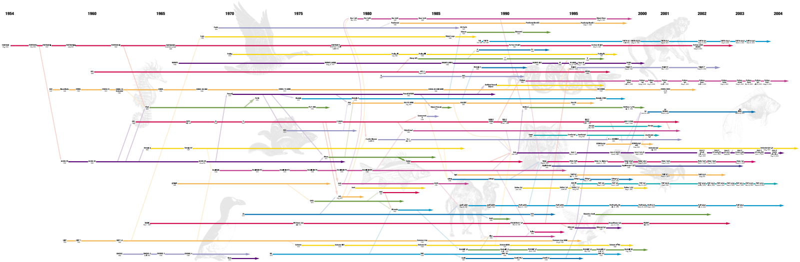
Programming Languages



Programming Languages



How we can classify all these languages?

Programming paradigms

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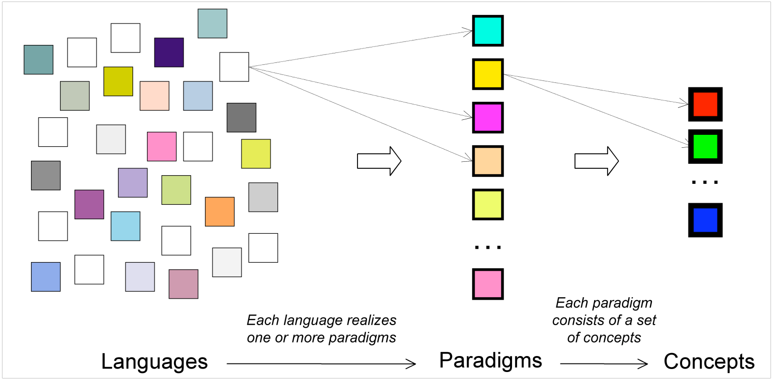
Solving a programming problem requires choosing the right concepts!

Programming paradigms



Each language realizes one or more paradigms

Each paradigm consists of a set of concepts

Languages ⟶ Paradigms ⟶ Concepts

Programming paradigms

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- functional: computation is treated as the evaluation of mathematical functions and avoids changing-state and mutable data.
- declarative/logical: expresses the logic of a computation without describing its control flow. A program consists in a set of sentences in logical form, expressing facts and rules about some problem domain.

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- declarative/logical: expresses the logic of a computation without describing its control flow. A program consists in a set of sentences in logical form, expressing facts and rules about some problem domain.
- object-oriented: it is based on the concept of objects, which may contain both data, the fields, and code, the methods.

This lecture. . .

In this lecture. . .

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...we will first introduce basic notions of functional programming...

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. . . finally an overview of modern programming languages is provided.

Functional programming in F#: Basic Concepts

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Functional programming

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Functions are first-class values and can be assigned to names (*variables*).

Computations consist in the appropriate compositions of defined functions.

We will consider F#, a modern functional language integrated in the .Net framework.

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F# (pronounced *F sharp*)...

... is a strongly typed programming language;

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Here we will main consider the functional aspects!

F# programming language

Primitive Types (1/2)

- `bool`, Boolean values (true or false).
- `byte`, Unsigned byte (from 0 to $2^8 - 1$).
- `sbyte`, Signed byte (from -2^7 to $2^7 - 1$).
- `int16`, 16-bit integer (from -2^{15} to $2^{15} - 1$).
- `uint16`, 16-bit integer (from 0 to $2^{16} - 1$).
- `int`, 32-bit integer (from -2^{31} to $2^{31} - 1$).
- `uint32`, 32-bit unsigned (from 0 to $2^{32} - 1$).
- `int64`, 64-bit integer (from -2^{63} to $2^{63} - 1$).
- `uint64`, 64-bit unsigned int (from 0 to $2^{64} - 1$).
- `char`, Unicode character values.
- `string`, Unicode text.
- `decimal`, Floating point data type that has at least 28 significant digits.

F# programming language

Primitive Types (2/2)

- `unit`, Indicates the absence of an actual value.
- `void`, Indicates no type or value.
- `float32`, A 32-bit floating point type.
- `float`, A 64-bit floating point type.

F# programming language

Values (1/2)

- `bool`: `true`, `false`.
- `byte`, an integer with postfix `y` (`86y`).
- `sbyte`, an integer with postfix `uy` (`86uy`).
- `int16`, an integer with postfix `s` (`86s`).
- `uint16`, an integer with postfix `us` (`86us`).
- `int`, an integer with the optional postfix `l` (`86` or `86l`).
- `uint32`, an integer with postfix `u` or `ul` (`86u` or `ul`).
- `int64`, an integer with postfix `L` (`86L`).
- `uint64`, an integer with postfix `UL` (`86UL`).
- `char`, a single symbol surrounded by single quotes (`'a'`).

F# programming language

Values (2/2)

- string, can be:
  - ... a sequence of characters surrounded by double quotes ("Hello\n\n World!");
  - ... a sequence of characters surrounded by double quotes and prefixed with @ (@"Hello\n\n World!");
  - ... a portion of text (possibly on multiple lines) surrounded by """

```
"""Hello

World!"""
```

- decimal, a floating point value postfixed with M (0.35M).
- unit, the value ().
- float32, a floating point postfixed with f or F (0.35f or 035F).
- float, a floating point in decimal or exponential form (0.35 or 3.5E−1).

F# programming language

Basic concepts

Basic construct in F# is `let` that can be used to associate a name with a value

```
let num = 10
let str = "F#"
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Basic concepts

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```
let num = 10
let str = "F#"
```

Each name has a type that is inferred from the associated expression!

Above:

- `num` has type `int`;
- `str` has type `string`;

F# programming language

Operators

Arithmetic Operators: +, −, ∗, /, %, ∗∗;

Comparison Operators: =, <, <=, >, >=, <>;

Boolean Operators: not, ||, &&;

Bitwise Operators: &&&, |||, ^^^, ~~~, <<<, >>>;

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Arithmetic and Comparison operators are overloaded: the exact type depends on the type of their argument!

Differently from `Java`, no implicit cast is done!

F# programming language

Simple type errors!

```
let x = 86u //x has type ubyte
let y = 86  //y has type int

let z = x+y //This is an error!!!!
```

F# programming language

Basic concepts

Functions are first-class values and can be associated with names as any other built-in types:

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let f1 = fun x -> x+1

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let f3(x,f) = f(x+2)+1

let y = f3(1,f)
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Function types have the form: `type1 -> type2`

F# programming language

Type inference

The idea of type inference is that you do not have to specify the types of F# constructs except when the compiler cannot conclusively deduce the type.

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This does not mean that F# is a dynamically typed language or that values in F# are weakly typed.

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F#, like almost all functional languages, is statically typed!

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Type annotations can be used to help the compiler to infer the expected type.

F# programming language

Type inference

```
//No annotation, inferred type: int*int -> int
let f(x,y) = x+y
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F# programming language

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```
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//Parameter x is annotated as float, inferred type: float*
    float -> float
let f(x: float, y) = x+y
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    //Name x is annotated as float, inferred type: float*
    float -> float
let f(x,y) = (x: float)+y


    //Return type of f is float,
    //    inferred type: float*float -> float
let f(x,y):float = x+y
```

F# programming language

Partial evaluation

Let us consider the following functions:

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F# programming language

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F# programming language

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val f1 : x:int * y:int -> int
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F# programming language

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F# programming language

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The two approaches are in fact equivalent! The second one is the standard (and more efficient).

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The following type is inferred for function `select`:

```
val select : x:'a -> y:'a -> z:'b -> w:'b -> 'b
    when 'a : equality
```

F# programming language

Recursive functions. . .

In functional programming the use of recursive definition is crucial.

F# programming language

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```
let fib x =
      if x<1 then
         1
      else
         (fib x-1)+(fib x-2)
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```
let rec fib(x) =  //Note here the use of 'rec'
      if x<=2 then
         1
      else
         (fib x-1)+(fib x-2)
```

F# programming language

Tuples...

A tuple is a grouping of unnamed but ordered values, possibly of different types.

```
(element , ... , element)
```

F# programming language

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(element, ... , element)
```

Example:

```
let fib(x) =
    let rec _fib(x) =
        if x<=2 then
            (1,1)
        else
            let (a,b)=_fib(x-1)
            in
                (a+b,a)
    in
        let (a,_) = _fib(x)
        in
            a
```

F# programming language

Lists. . .

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A list in F# is an ordered, immutable series of elements of the same type. Lists have type `'a list`.

You can define a list by explicitly listing out the elements, separated by semicolons and enclosed in square brackets;

```
let list123 = [ 1; 2; 3 ] //Type int list
let emptylist = [] //Type 'a list!
```

F# programming language

Lists. . .

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```
let list123 = [ 1; 2; 3 ] //Type int list
let emptylist = [] //Type 'a list!
```

List operations:

- `::` is used to add an element at the beginning of the list: `a :: list1`
- `@` is used to concatenate two lists: `l1@l2`

F# programming language

Lists. . .

F# programming language

Lists. . .

You can also define list elements by using a range indicated by integers separated by the range operator ..:

```
let list1 = [ 1..10 ]
```

List can be generated in a symbolic way expressions:

```
   let fiblist = [ for i in 1 .. 10 -> fib(i) ];;
```

Pattern matching. . .

The `match` expression provides branching control that is based on the comparison of an expression with a set of patterns.

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// Match expression.
match test-expression with
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Conditions are boolean expressions that can be used to limit the selection.

Example: Polynomial evaluation

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$$a_n x^n + \cdots + a_1 x + a_0$$

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let poly = [ an; ... a1; a0 ]
```

Write a function `eval` that received in input a list of coefficients and a value `x` computes the value of the polynomial.

Example: Polynomial evaluation

Solution 1:

```
let rec eval clist (x: float) =
        match clist with
        | [] -> 0.0
        | c::tail -> c*(x**float(clist.Length-1))+(eval tail
   x)
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Example: Polynomial evaluation

Solution 1:

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```

Solution 2:

```
let eval2 clist (x: float) =
        let rec _eval2 clist v =
            match clist with
            | [] -> v
            | c::tail -> _eval2 tail (v*x+c)
        in
            _eval2 clist 0.0
```

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Find the first element in a list matching a predicate:

```
let rec findFirstMatching pred l =
    match l with
    | [] -> None
    | v::tail -> if (pred v) then Some v
                 else findFirstMatching pred tail
```

Option type. . .

The option type is used when an actual value might not exist for a named value or variable.
An option has an underlying type and can hold a value of that type, or it might not have a value

```
'a option
```

Find the first element in a list matching a predicate:

```
let rec findFirstMatching pred l =
   match l with
   | [] -> None
   | v::tail -> if (pred v) then Some v
                else findFirstMatching pred tail
```

The type of `findFirstMatching` is:

```
pred:('a -> bool) -> l:'a list -> 'a option
```

Custom data type: Discriminated Unions...

Discriminated unions provide support for values that can be one of a number of named cases, possibly each with different values and types:

```
type type−name =
    | case−identifier1 [of [ fieldname1 : ] type1 [ ∗ [
    fieldname2 : ] type2 ...]
    | case−identifier2 [of [fieldname3 : ]type3 [ ∗ [
    fieldname4 : ]type4 ...]
```

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Discriminated unions provide support for values that can be one of a number of named cases, possibly each with different values and types:

```
type type-name =
    | case-identifier1 [of [ fieldname1 : ] type1 [ * [
   fieldname2 : ] type2 ...]
    | case-identifier2 [of [fieldname3 : ]type3 [ * [
   fieldname4 : ]type4 ...]
```

Example:

```
type Shape =
    | Rectangle of width : float * length : float
    | Circle of radius : float
    | Prism of width : float * float * height : float
```

Example: Binary Search Trees!

Binary search trees keep their keys in sorted order:

- elements are inserted/removed from the tree by following the principle of binary search;
- elements traverse the tree from root to leaf by making decisions on the base of comparison.

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- elements traverse the tree from root to leaf by making decisions on the base of comparison.

Exercise:

1. develop a data type for BST;
2. implement basic operations on BST...
    - insertion;
    - search;
    - deletion.

Example: Binary Search Trees!

Data type:

```
type bstree =
    EMPTY
  | NODE of value: int * left: bstree * right: bstree
```

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type bstree =
    EMPTY
  | NODE of value: int * left: bstree * right: bstree
```

Add a value in the tree:

```
let rec add v t =
    match t with
    | EMPTY -> NODE(v,EMPTY,EMPTY)
    | NODE(v1,l,r) when v1<v -> NODE(v1,l,add v r)
    | NODE(v1,l,r) -> NODE(v1,add v l, r)
```

Example: Binary Search Trees!

Search for an element:

```
let rec contains v t =
    match t with
    | EMPTY -> false
    | NODE(v1,_,_) when v1=v -> true
    | NODE(v1,l,r) when v1<v -> contains v r
    | NODE(v1,l,r) -> contains v l
```

Example: Binary Search Trees!

Search for an element:

```
let rec contains v t =
    match t with
    | EMPTY -> false
    | NODE(v1,_,_) when v1=v -> true
    | NODE(v1,l,r) when v1<v -> contains v r
    | NODE(v1,l,r) -> contains v l
```

Merging trees:

```
let rec merge t1 t2 =
    match t1,t2 with
    | EMPTY,_ -> t2
    | _,EMPTY -> t1
    | NODE(v1,l1,r1),NODE(v2,l2,r2) when v1<v2 ->
            NODE(v1,l1,merge r1 t2)
    | NODE(v1,l1,r1),NODE(v2,l2,r2) ->
            NODE(v2,l2,merge r2 t1)
```

Example: Binary Search Trees!

Removing an element:

```
let rec remove v t =
    match t with
    | EMPTY -> EMPTY
    | NODE(v1,l,r) when v1=v -> merge l r
    | NODE(v1,l,r) when v1<v -> NODE(v1,l,remove v r)
    | NODE(v1,l,r) -> NODE(v1,remove v l,r)
```

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The type `bstree` can only contain integer values.

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We can use Generics!

Generics. . .

Generic programming is a style of computer programming in which elements of a program (procedures, functions, data-types,. . .) are written in terms of types to-be-specified-later.

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In F# function values, methods, properties, and aggregate types such as classes, records, and discriminated unions can be generic.

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Generic functions and types enable you to write code that works with a variety of types without repeating the code for each type:

```
/ Explicitly generic function.
let function-name<type-parameters> parameter-list =
function-body

// Explicitly generic type.
type type-name<type-parameters> type-definition
```

Generics. . .

Automatic Generation: Type Inference

The F# compiler, when it performs type inference on a function, determines whether a given parameter can be generic.

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Example:

```
let max a b = if a > b then a else b
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This function has type `'a -> 'a -> 'a when 'a comparison`.

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Example:

```
let max a b = if a > b then a else b
```

This function has type `'a -> 'a -> 'a when 'a comparison`.

Above `when 'a comparison` is a constraint.

Example...

We can change the definition of bstree as follows:

```
type bstree<'T when 'T: comparison> =
  EMPTY
  | NODE of value: 'T * left: 'T bstree * right: 'T bstree
```

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We can change the definition of `bstree` as follows:

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type bstree<'T when 'T: comparison> =
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  | NODE of value: 'T * left: 'T bstree * right: 'T bstree
```

We have not to change the functions `add`, `contains`, and `remove`!

Namespace. . .

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```
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```

If you want to put code in a namespace, the first declaration in the file must declare the namespace. The contents of the entire file then become part of the namespace.

Namespaces cannot directly contain values and functions. Instead, values and functions must be included in modules, and modules are included in namespaces. Namespaces can contain types, modules.

Modules

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module [accessibility-modifier] module-name =
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Grouping code in modules helps keep related code together and helps avoid name conflicts in your program.

```
module [accessibility-modifier] module-name =
    declarations
```

We can build the module of Bstrees!

Modules: List...

Modules: List. . .

average: Returns the average of the elements in the list.

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```
// Signature:
List.average : ^T list -> ^T
  (requires ^T with static member (+)
    and ^T with static member DivideByInt
    and ^T with static member Zero)

// Usage:
List.average list

// Example
average([1.0 .. 10.0])
```

Modules: List...

averageBy: Returns the average of the elements generated by applying the function to each element of the list.

```
// Signature:
List.averageBy : ('T -> ^U) -> 'T list -> ^U
  (requires ^U with static member (+)
    and ^U with static member DivideByInt
    and ^U with static member Zero)

// Usage:
List.averageBy projection list


// Example
List.averageBy (fun x -> x**2.0) [ 1.0 .. 10.0 ];;
```

Modules: List. . .

filter : Returns a new collection containing only the elements of the collection for which the given predicate returns true.

```
// Signature:
List.filter : ('T -> bool) -> 'T list -> 'T list

// Usage:
List.filter predicate list

// Example
List.filter (fun x -> x%3=0) [ 1 .. 100 ];;
```

Modules: List. . .

map: Creates a new collection whose elements are the results of applying the given function to each of the elements of the collection.

```
// Signature:
List.map : ('T -> 'U) -> 'T list -> 'U list

// Usage:
List.map mapping list

// Example
List.map (fun x -> x*x) [ 1 .. 10 ];;
```

Modules: List. . .

reduce: Applies a function to each element of the collection, threading an accumulator argument through the computation.

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Given a function `f` and a list containing `i0,i1,i2,...,ik` computes:

```
f (... (f i0 i1) i2 ...) ik
```

Modules: List. . .

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Given a function `f` and a list containing `i0,i1,i2,...,ik` computes:

```
    f (... (f i0 i1) i2 ...) ik
```

```
// Signature:
List.reduce : ('T -> 'T -> 'T) -> 'T list -> 'T

// Usage:
List.reduce reduction list

// Example:
List.reduce (fun x y -> x+y) [1..100]
```

Map-Reduce. . .

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- a filter that restricts the dataset to the elements satisfying a predicate;

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The Map-Reduce patter relies on three main functions:

- a filter that restricts the dataset to the elements satisfying a predicate;
- a map function that processes elements dataset;
- a reduce function that combines result.

Excercises. . .

Ex. 0: Download and install F# developing environment. See instructions available here:

`https://docs.microsoft.com/en-us/dotnet/fsharp/get-started/`

Ex. 1: Write a function that given input *a* and *b* computes their *mcd*.

Ex. 2: Write a function that given an input *n* returns `true` if *n* is a *prime number* and `false` otherwise.

Ex. 3: Write a function that given in input an integer *n* computes the list of its prime factors.

Ex. 4: Can *map-filter-reduce* be used to simplify the code we have considered so far?

BS Trees. . .

Ex. 5 Implement function `size` that given a tree `t` computes the number of elements stored in `t`.

Ex. 6 Implement function `height` that given a tree `t` computes its height (an empty BST has height equal to 0).

Ex. 7 Implement function `balance` that given a tree `t` computes a tree `t1` with the same elements its height (an empty BST has height equal to 0).

Ex. 8 Implement AVL data structure.

To be continued. . .